**1** The diagram below shows the curve . The curve has a stationary point of inflexion at  and a maximum point at . The curve also has vertical asymptotes ,  and a horizontal asymptote .

*x*

*y*

0











Sketch the curve  labelling clearly the coordinates of any axial intercepts, turning points and equations of asymptotes where applicable. [3]

**2** A local family consisting of adults, children and senior citizens, is planning for a trip to the Jewel Bird Park. The ticket prices are listed as shown below. If all members in the family purchase the Local Resident Discounted tickets, they will need to pay a total price of $301.20. If all members in the family purchase the Wildlife Quest Bundle, they will need to pay a total price of $478.50. Given that the number of adults is four times the number of senior citizens, find the number of adults, children and senior citizens in the family.[3]

|  |  |
| --- | --- |
| **Local Resident Discounted Tickets**  **Adult $27.00**  **Senior Citizen $15.00**  Ages 60 and above  **----------------------------------------------**  **Child $18.40**  Ages 3 to 12 | **Wildlife Quest Bundle**  **Adult Admission + $39.00**  **Wildlife Quest**  **-------------------------------------------**  **Child Admission + $29.50**  **Wildlife Quest**  Ages 3 to 12 |

**3** A curve *C* has equation , where is a positive constant.

1. Express  in terms of  and . [3]
2. Explain why there is no point on *C* where the tangent is parallel to the *x*-axis. [2]

**4** With respect to origin *O*, the distinct points *P*, *Q*, *R* and *S* have position vectors , ,  and  respectively. It is known that  is a unit vector.

1. Given that , state, with justification, the relationship between the points *P*, *Q* and *R*. [2]
2. Give a geometrical interpretation of . [1]
3. Given that  and that is parallel to and in the opposite direction of , find . [2]

**5** A curve has equation .

1. The region *R* is bounded by the curve, the lines ,  and the *y*-axis. Find the exact area of *R*. [4]
2. Find the volume generated when *R* is rotated about the *x*-axis through 360, giving your answer correct to 3 decimal places. [3]

**6** The sum of the first *n* terms of a series is given by the expression .

1. State the first term of the series in terms of e. [1]
2. By finding the *n*th term of the series, show that this is a geometric series. [3]
3. Explain why the sum to infinity, *S*, of the series exists, and determine the exact value of *S*. [3]

**7** A curve *C* has parametric equations

 ,  where .

1. Find the exact values of  at which *C* crosses the *x*-axis. [1]
2. Sketch *C*, labelling the coordinates of the points at which *C* crosses the *x*-axis. [2]
3. Find the equation of the tangent to *C* at the point *P* with parameter *p*, where. [3]
4. *Q* is a point on *C* such that the tangent at *Q* is parallel to the *y*-axis. Find the area bounded by *C*, the tangent at *Q* and the *x*-axis. [3]

**8** A function f is said to be self-inverse if for all *x* in the domain of f.

It is given that g is a self-inverse function and is defined by

, for  ,

where *a* and *b* are constants and 

1. Find the value of *b* and show that . [3]
2. Find  [1]

The function h is defined by



1. Given that  exists, find  in similar form. [4]
2. Show that gh exists and find the exact range of gh. [3]

**9** The curve  has equation 

1. Find the value of *x* when . [1]
2. Sketch *C*, showing clearly the equations of any asymptotes and coordinates of any turning points and axial intercepts. [3]
3. Solve the inequality [1]



1. Using the result in part **(iii)**, solve the inequality [2]



1. Find the area bounded by the curve *C* and the *x*-axis, leaving your answer in the form , where *a*, *b* and *c* are integers. [5]



**10** A perfume maker designs a prototype of a perfume bottle of fixed volume for a new fragrance as shown in the diagram below.







*O*

*Y*

The prototype comprises 2 different segments where the vertical axis of the prototype,, is . The bottom segment is a glass cylinder of radius *r* cm and height *h* cm. The top segment is a chrome-plated plastic hemisphere of radius cm. It is assumed that the prototype is of negligible thickness and there is no gap between the 2 segments.

1. Find *h* in terms of *V*, *r* and . [2]
2. Given that the cost of manufacturing the glass cylinder is $8 per  and the chrome-plated plastic is $3 per , show that the total cost of manufacturing the prototype is . Hence, using differentiation, find the exact value of  such that the cost of manufacturing the prototype is minimum. [7]

It is now given that the diameter of the chrome-plated hemisphere is 3 cm and the height of the glass cylinder is 5 cm.

1. A crack at the bottom of the prototype causes the perfume to leak out of the prototype at a constant rate of . Given that perfume is initially filled to the brim of the top segment of the prototype, find the exact rate of decrease of the height of the perfume in the prototype 5 seconds after the prototype has cracked. [3]

**11** The points *A*, *B* and *C* have coordinates ,  and  respectively. The line  has equations  and the line  passes through *A* and *B*.

1. Find the coordinates of the foot of perpendicular from *C* to . [4]
2. Find the acute angle between  and . [3]
3. The point *D* is on  such that the distance from *D* to *A* is twice the distance from *D* to *B*. Find the possible point(s) *D*. [4]
4. The line  passes through point  and is perpendicular to both  and . Find the equation of . [2]

**12** A couple takes up a housing loan of $*L* and the interest is charged before each monthly repayments at a fixed rate of *p*% per annum. Their monthly repayment commences on 1 September 2021. Monthly repayments of $*x* are due and payable on the first day of subsequent months until their housing loan is fully repaid.

1. State an expression in terms of *L* and *p* for the interest charged before their first repayment on 1 September 2021. [1]
2. Show that the outstanding loan at the start of the *n*th month after their monthly repayment is given by

.

[3]

The couple is taking a housing loan of $504,000 at a fixed interest rate of 2.6% per annum.

1. Calculate the monthly repayment if the couple plans to repay the loan in 30 years. [2]
2. Given that the couple decides to pay monthly repayments of $4000, find the date at which the couple will be able to fully repay their housing loan and the amount that the couple pays for their final monthly repayment. [3]

The couple decides to start adopting a savings plan on 1 September 2021. The couple decides to deposit $*k* on 1 September 2021 to the savings plan and for each subsequent month, they will deposit $*a* more than the previous month. Each month, the savings plan gives a fixed interest of 0.1% for the amount deposited for that month. The total amount that the couple will have in the savings plan after *n* months is given by

.

1. Find the values of *k* and *a*. [2]
2. Assuming that the couple intends to pay monthly repayments of $4000 for their housing loan, find the least number of months that is needed so that the couple can use the amount in their savings plan to make a one-time repayment to fully repay their outstanding housing loan. [2]